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Short Note

Potential resonant screening effects on stellar ^{12}C + ^{12}C reaction rates

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Abstract. The $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion cross-sections show a resonant behavior down to the lowest energies accessible so far in the laboratory. If this tendency continues into the astrophysical energy range, the stellar $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ reaction rates have to be corrected for resonant screening effects, in addition to the conventional screening corrections. We estimate the resonant screening effects in the weak electron screening limit for hydrostatic burning and white-dwarf environments.

PACS. 26.50.+x Nuclear physics aspects of novae, supernovae, and other explosive environments – 21.30.-x Nuclear forces

Nuclear reactions are the energy source for many astrophysical objects [1]. When applied in the stellar environment, however, the nuclear-reaction rate, determined from measurements for bare nuclei (for electron screening corrections to low-energy laboratory cross-sections, see [2]), has to be corrected for plasma effects. The medium induces a screening potential $V_{\mathrm{sc}}(r)$ which effectively reduces the Coulomb barrier between the two colliding nuclei and enhances the reaction cross-section. This effective increase of the in-medium cross-section over the laboratory cross-section is conventionally described by an enhancement factor f_{s}. If $V_{\mathrm{sc}}(r)$ is approximately constant during the penetration process, *i.e.* for radii smaller than the outer barrier turning point, the potential can be replaced by the screening energy $U_0 = -V_{\mathrm{sc}}(0)$. Such a situation occurs in astrophysical sites in which the average Coulomb energy between neighboring ions is much less than the thermal energy of the plasma. As has been shown by Salpeter [3], in this *weak screening* limit the Debye-Hückel theory is applicable leading to an enhancement of the thermally averaged nuclear-reaction rate in the plasma. In cold ($T \sim 0$), but dense plasma, fusion reactions are induced by density fluctuations. Again, the medium-induced screening potential strongly enhances the reaction cross-sections. The appropriate formalism is developed for example in [4].

Besides this global screening enhancement an additional plasma correction to the nuclear-reaction rate can occur if the rate is dominated by narrow resonances. The general theory has been outlined in [4], but no application has been identified so far. We will point out in this note that this situation may occur for the $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion reaction, where the cross-sections show noticeable resonant structures down to the lowest energies measured so far in the laboratory ($E \sim 2.4$ MeV, [5]). Until now, the resonant structure of the $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion cross-section has been ignored in astrophysical applications and the respective reaction rate has been determined as a smooth average over the resonant contributions. Consistent with this procedure only the global plasma screening enhancement has been considered. However, if the resonant structure continues to even lower energies and the astrophysical reaction rate is due to the contributions of narrow resonances, one then has to consider that the entrance channel width of these resonances will be modified in the plasma. This resonant screening effect will reduce the reaction rate. Our discussion is based on the fact that for $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ resonances far below the height of the Coulomb barrier, the entrance channel width (Γ_a) is much smaller than the total resonance width. The latter (which is of order ~ 100 keV for the observed resonances above 2.4 MeV) is also noticeably smaller than the resonance energy. Hence, we can treat the contribution of these resonances to the thermally averaged reaction rate by the formalism developed for narrow, isolated resonances (*e.g.*, [1]), which for the

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$^{12}\mathrm{C} + {}^{12}\mathrm{C}$ system is

$$\langle \sigma v \rangle_R = \left(\frac{2\pi}{\mu k T} \right)^{3/2} \hbar^2 2(2J+1) \Gamma_a(E_\mathrm{r}) \exp\left(-\frac{E_\mathrm{r}}{kT} \right) , \tag{1}$$

where μ is the reduced mass and E_r and J are the energy and total angular momentum of the resonance.

In the medium the Coulomb potential $V_\mathrm{c}(r)$ has to be replaced by $V_\mathrm{c}(r) + V_\mathrm{sc}(r)$ ($V_\mathrm{sc}(r) < 0$). This modification will change the resonance energy in the plasma to $E'_\mathrm{r} < E_\mathrm{r}$. Relatedly, the resonance width in the plasma $\Gamma'(E'_\mathrm{r})$ is proportional to the penetration factor [6]

$$\Gamma'(E'_\mathrm{r}) \sim \exp\left(-2\sqrt{\frac{2\mu}{\hbar^2}} \int_{r_\mathrm{in}}^{r_\mathrm{out}} \sqrt{(V_\mathrm{c}(r) + V_\mathrm{sc}(r) - E'_\mathrm{r})} \mathrm{d}r \right) , \tag{2}$$

where r_in and r_out are the inner and outer barrier turning points, respectively, and μ the reduced mass of the two colliding nuclei. The situation simplifies, if one has $V_\mathrm{sc}(r) = V_\mathrm{sc}(0) = -U_0$ for radii $r < r_\mathrm{in}$. Then, in a good approximation one has $E'_\mathrm{r} = E_\mathrm{r} - U_0$. In the weak screening limit one even has $V_\mathrm{sc}(r) \approx V_\mathrm{sc}(0)$ for $r < r_\mathrm{out}$, resulting in $\Gamma'(E'_\mathrm{r}) = \Gamma(E_\mathrm{r})$ Such a situation holds in a massive star ($\sim 20 M_\odot$), where carbon core burning occurs at temperatures around 10^9 K and for densities of order $\rho = 2 \cdot 10^5$ g/cm^3 [1]. Thus no resonant screening correction for the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction cross-section is expected in hydrostatic carbon burning.

In passing we mention that many nuclear reactions occuring in explosive hydrogen burning [7] are dominated by narrow resonances, *e.g.*, the $^{15}\mathrm{O}(\alpha,\gamma)^{19}\mathrm{Ne}$ reaction which is usually believed to be a key for the break-out of the matter-flow from the hot CNO-cycle in novae or X-ray bursters. However, in both sites the weak screening conditions approximately hold and no resonant screening corrections have to be applied.

In general the magnitude of $V_\mathrm{sc}(r)$ decreases with increasing radius; at large distances the medium-induced screening potential cancels the Coulomb repulsion between the colliding nuclei, *i.e.* $V_\mathrm{c}(r) + V_\mathrm{sc}(r) = 0$ for large r. If $V_\mathrm{sc}(r)$ already decreases for radii r under the barrier ($r_\mathrm{in} < r < r_\mathrm{out}$) the width of the barrier to be penetrated in the medium is longer than the barrier width for bare nuclei. Correspondingly one has for the resonance width $\Gamma'(E'_\mathrm{r}) < \Gamma(E_\mathrm{r})$ which leads to an additional screening correction of the resonant cross-section which we like to denote by $f_\mathrm{r} = \frac{\Gamma'(E'_\mathrm{r})}{\Gamma(E_\mathrm{r})}$. The total screening correction is then given by $f_\mathrm{tot} = f_\mathrm{s} \cdot f_\mathrm{r}$. Obviously the resonant screening correction reduces the cross-section in the medium, counteracting the overall enhancement by f_s.

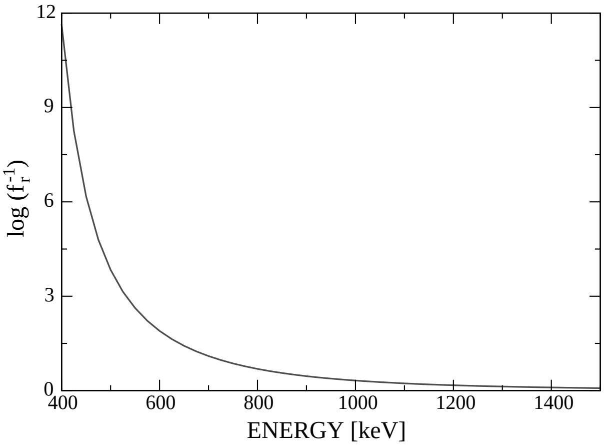

Fig. 1. The inverse of the resonant screening correction factor f_r^{-1} for the stellar $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction rate as function of resonance energy in the entrance channel. The calculation has been performed for a white-dwarf environment [8] ($T = 5 \cdot 10^7$ K, $\rho = 2 \cdot 10^9$ g/cm^3).

For reactions in cold, dense plasma Salpeter and van Horn have derived an appropriate analytical expression for the resonant screening correction. This description is appropriate for a carbon white-dwarf environment with $T = 5 \cdot 10^7$ K and $\rho = 2 \cdot 10^9$ K [8]. Following [4] one then finds a resonant screening correction factor for the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction if this reaction indeed proceeds via a narrow resonance at energy E_r:

$$f_\mathrm{r} = \exp\left(-\frac{\pi}{\sqrt{\lambda \epsilon}} \left[\frac{1.22}{\epsilon^3} - \frac{3.1}{\epsilon^6} + \frac{75}{\epsilon^9} \right] \right) \tag{3}$$

with the abbreviations

$$\lambda = \frac{1}{432} \left(\frac{\rho_{12}}{1.629} \right)^{1/3} , \quad \epsilon = \frac{1}{36} \left(\frac{1.629}{\rho_{12}} \right)^{1/3} \frac{E_\mathrm{r}}{49.6} , \tag{4}$$

where ρ_{12} measures the density in 10^{12} g/cm^3 and the resonance energy E_r is defined in keV. Obviously the correction becomes more important with increasing density and at lower resonance energies.

Although the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ resonance energies below $E \leq 2.4$ MeV are currently not known, we will now assume that the resonance structures observed in the fusion data at $E \geq 2.4$ MeV continue to lower energies. While this will probably not strongly affect the smooth energy dependence of the averaged astrophysical S-factor derived from the data at higher energies and used for the reaction rate in astrophysical applications so far, it will, however, change the screening corrections to the rate in the astrophysical environment. We will estimate the reduction of the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction rate due to the resonant screening correction factor f_r for carbon white-dwarf environment hypothetically assuming a resonance in the energy interval 0.4–2 MeV.

As is shown in fig. 1 the resonant screening correction can lead to a significant reduction of the reaction rate, which amounts to more than 11 orders of magnitude if the resonance energy is as low as $E_\mathrm{r} = 400$ keV. Such a change might influence the carbon ignition density in white dwarfs. We note, however, that despite such a potentially large correction global screening still dominates, which due to table 1 of [8] amounts to an enhancement of the rate by 12 orders of magnitude. But this estimate might also change if the short-ranged potential which gives rise to the resonances is taken into account [9].

In summary, the $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ reaction rate in a white-dwarf environment might change significantly, if the reaction is dominated by narrow resonances at the relevant energies. This can only be established experimentally. If such a resonance is indeed being observed it is desirable to study the plasma enhancement factor due to many-nuclear correlations including a nuclear potential which supports resonances at low $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ scattering energies.

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